

# STUDY OF FUZZY LOGIC WITH THEIR APPLICATIONS

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**Abstract** - This study will investigate fuzzy logic as well as its many applications. In addition to this, we investigate the characteristics of a number of other fuzzy logics. The author presents a concise explanation of the fundamental concepts underlying fuzzy logic, in addition to a number of different instances of its applications. The basic laws of inference, the meaning representation and inference, the linguistic variable, and its application in fuzzy control are all discussed in this article. Fundamentally, the objective of the fuzzy control systems theory is to develop a number of novel and specialized techniques and methods, as well as to extend the already successful conventional control systems techniques and methods to a significantly larger class of complicated, convoluted, and inadequately represented systems, which are known as fuzzy systems. The extent to which a person's health may be considered an essential measure of human development is directly proportional to that person's potential to lead a life that is both socially and economically useful. This chapter examines the use of multiplication and addition in Sanchez's

medical diagnostic methodology by applying two distinct algorithms to the same case. The goal of this investigation is to better understand how Sanchez's method works.

In this paper, we discuss a method based on fuzzy logic that may be used to solve nonlinear optimization problems that also include linear approximation relationships. The nonlinear optimization issue is converted into a linear optimization problem or a linear programming problem by the suggested technique, which linearly approximates the nonlinear objective function as well as the constraint functions that define the feasible regions around an initial feasible point.

**Key Words:** optimization problems, fuzzy logic, Health, linear approximation

## 1. INTRODUCTION

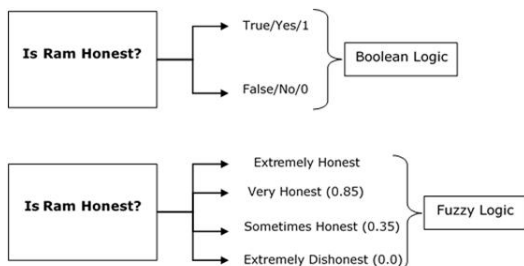
It is claimed that something is fuzzy if it cannot be determined or if it causes

confusion. Because any event, process, or function that is always changing cannot always be defined as true or untrue, we have little choice but to characterize activities of this kind in a hazy manner.

### ➤ What is Fuzzy Logic?

The process through which people arrive at conclusions is modeled by fuzzy logic. It can deal with data that is unclear and perplexing. This is an evident simplification of issues that arise in the actual world, and it uses degrees of truth rather than the more traditional true/false or 1/0 Boolean logic.

Take a look at the sample that follows. It discusses how values in fuzzy systems are represented by integers ranging from 0 to 1, which might be confusing to certain people. In this scenario, the value 1.0 denotes the steadfast truth, whereas the value 0.0 denotes the unwavering deception. The truth value of a fuzzy system is the integer that best represents the value of the system.



**Figure 1: Fuzzy System**

To put it another way, fuzzy logic is not reasoning that is cloudy in and of itself; rather, it is logic that is used to characterize fuzziness. It is highly possible that there are a great number of other instances similar to this one that will assist us in comprehending fuzzy logic.

Two-value logic is generalized into fuzzy logic, which is a logical framework that may be used for reasoning in uncertain contexts. A fuzzy set is a class of items in which membership is decided by degree rather than by clearly defined boundaries. In this computing and reasoning system, they play an important role [Yen and Langari, 1999]. The mathematical foundation of fuzzy logic is tied to fuzzy set theory, which is an extension of classical set theory. This means that fuzzy logic may be thought of as an extension of classical set theory.

The first argument in favor of fuzzy logic was provided by Lotfi A. Zadeh of the University of California, Berkeley, in the year 1965. When Zadeh realized that normal computer logic could not be used to handle data that reflected arbitrary or foggy conceptions, he developed fuzzy logic. This enables computers to distinguish between data using a spectrum of grayscales, similar to how humans think. Zadeh's discovery was groundbreaking. In 1973, he was the first person to bring the idea of linguistic variables to the scientific community. As a consequence of this, the performance of electromechanical

controllers need to be improved by imitating the way in which this sort of information is handled. In the beginning, it took some time to develop the idea, but by the beginning of the 1970s, it had caught the attention of a small number of scientists from all over the world. There were some Westerners and Japanese engineers among them, but the majority of the Westerners were mathematicians.

Fuzzy logic has been used in a wide range of processes and products, including those used in manufacturing, consumer electronics, transportation systems, vehicles, and home appliance manufacturing. There are currently a great number of consumer goods accessible in Japan that were made employing fuzzy technology; some of these goods are now available in the United States and Europe.

#### ➤ Fuzzy Qualifiers

Let us now understand Fuzzy Qualifiers. A Fuzzy Qualifier is also a proposition of Fuzzy Logic. Fuzzy qualification has the following forms –

#### ➤ Fuzzy Qualification Based on Truth

It claims the degree of truth of a fuzzy proposition.

**Expression** – It is expressed as  $x$  is  $t$ . Here,  $t$  is a fuzzy truth value.

**Example** – (Car is black) is NOT VERY True.

#### ➤ Fuzzy Qualification Based on Probability

It claims the probability, either numerical or an interval, of fuzzy proposition.

**Expression** – It is expressed as  $x$  is  $\lambda$ . Here,  $\lambda$  is a fuzzy probability.

**Example** – (Car is black) is Likely.

#### ➤ Fuzzy Qualification Based on Possibility

It claims the possibility of fuzzy proposition.

**Expression** – It is expressed as  $x$  is  $\pi$ . Here,  $\pi$  is a fuzzy possibility.

**Example** – (Car is black) is Almost Impossible.

#### ➤ Fuzzy Inference System

The principal function of the fuzzy inference system, which is one of the components that comprise a fuzzy logic system, is primarily decision-making. It does this by using rules of the form "IF...THEN" in conjunction with connectors of the kind "OR" or "AND."

#### ➤ Characteristics of Fuzzy Inference System

- The following is a list of some FIS features:
- The output of the FIS is always a fuzzy set, regardless of how crisp or fuzzy the input to the FIS was;
- When the FIS is employed as a controller, the output must be fuzzy.
- In order to transform fuzzy variables into more precise ones,

S. No	Exam p1	Exam p2	Performance	S. No	Ep-1	Ep-2	Performance
1	35	70	0.531	11	87	99	0.908
2	27	40	0.276	12	50	27	0.431
3	60	85	0.8	13	68	48	0.684
4	07	30	0.174	14	38	45	0.431
5	48	68	0.684	15	22	52	0.344
6	42	58	0.573	16	32	60	0.516
7	50	64	0.659	17	100	100	0.937
8	44	52	0.516	18	53	78	0.72
9	40	24	0.248	19	82	61	0.901
10	74	68	0.729	20	57	43	0.562

the FIS would be equipped with a defuzzification unit.

## 2. EVALUATION OF STUDENTS PERFORMANCES USING FUZZY LOGIC

The performance of a fuzzy logic application has been evaluated based on the students' study results. Performance numbers and score tables are generated

based on the information given. To grasp it, the efficacy of the classical approach and fuzzy logic for the presented facts are compared.

Lotfi Zadh invented the fuzzy logic tool, a mathematical trading device, in 1965. It offers a soft computing interaction, which is an important concept in word processing. It provides a solution for dealing with granularity and imprecision. The fuzzy theory provides a method for describing linguistic ideas such as many, low, medium, and often few. In general, fuzzy logic provides an inference framework that supports the required human thinking abilities. In contrast, standard binary set theory specifies explicit events that either occur or do not occur. It calculates the possibility of a certain event occurring by applying probability theory concepts to a real-world occurrence.

### ➤ Applications of Fuzzy Logic in students' performance

The results of 20 different students on Exam Papers 1 and 2 are shown in Table 1. These results are comparable. The outcomes of both exams were fuzzy-fied for each individual student by using the

**Table 1: Calculated value of students' performance and exam paper scores**

membership functions described in Fuzzy Rules and Inference. The Mamdani fuzzy decision approach as well as the rule table are used in the process of determining the active

membership functions. After that, the center of the final geometric shape is computed, which helps to defuzz the performance value. This process is carried out once more for each student depending on the outcomes of the examination. Table 3 presents the results of the performance measurements as well as the paper grades for the selected data set.

The trapezoidal membership function was the same regardless of whatever inputs were used. Even if Exam Papers 1 and 2 were switched out, the computed performance value — for example, (48 and 68) and (68 and 48) — would not be affected in any way. If the time keeping or value range of the membership functions are not the same, then one of the tests will have a higher effect than the other on the result that is generated about the performance.

**Table 2: Calculate the Performance values and Exam Paper Scores**

S.No	Exam p1	Exam p2	Perf orm	S. No	Ep- 1	Ep- 2	Perf orm
1	35	70	0.542	11	87	99	0.591
2	27	40	0.259	12	50	27	0.865
3	60	85	0.627	13	68	48	0.784
4	07	30	0.200	14	38	45	0.500
5	48	68	0.784	15	22	52	0.490
6	42	58	0.44	16	32	60	0.5

			8				
7	50	64	0.519	17	100	100	0.33
8	44	52	0.745	18	53	78	0.758
9	40	24	0.719	19	82	61	0.64
10	74	68	0.47	20	57	43	0.292

Exam Paper 2 is structured to punish students for getting poor scores while rewarding those who do well. This scenario is shown in Table 2. As a result of this being the upper limit of the limit value, performance values went down for test scores that were lower than 50 and went up for test scores that were higher than 50.

### 3.FUZZY LOGIC APPLICATIONS TO SOME OPTIMIZATION PROBLEMS

#### ➤ Optimization Problems

We are tasked with the responsibility of optimizing a broad variety of professions and disciplines, including engineering design, regional politics, logistics, operations research, decision theory, and a great many more. Discovering the most effective answers is one of the most important objectives here. For the most part of the last several decades, the strategies and models that have been deployed in these fields have either been "crisp" or "hard," which means that solutions have either

been seen as attainable or not, above or below a given degree of ambition. This has led to a lack of progress in these areas. Methods and techniques of optimization, especially those of mathematical programming, have been utilized for a long time to effectively handle a broad variety of problems involving and connected to technological systems with reasonably well defined behavior and structure. This practice has been ongoing for a number of years. This has made it possible for optimization problems to be stated and solved using well-established, highly effective classical computational and analytical methodologies. These problems may include objective functions and constraints.

The use of spoken language presents a significant challenge to the use of conventional optimization strategies and models in complex optimization systems, such as those in which human judgments, preferences, and other such aspects often take the form of a fuzzy (imperfect) representation of the ideal solution. In the last couple of decades, fuzzy set theory and fuzzy logic (in its broadest senses) have seen significant

advancements. [Yen and Langari, 2003; Fang et al., 2013; Song et al., 2013] This progression illustrates that this theory offers a practical mechanism for the production of analytical processes that allow for the appropriate processing of "complex/soft" information in order to arrive at a realistic option. This is shown by the fact that this theory gives a practical mechanism for the creation of this progression.

This thesis makes use of fuzzy logic as a tool to overcome the challenges associated with building and analyzing complicated optimization systems. These systems include non-sharp borders, ideas, and information that are difficult to deal with using normal mathematical approaches. Fuzzy logic is used as a strategy to solve these issues. The use of fuzzy logic to solve complicated optimization issues, in our opinion, not only provides a method that is economical for simulating large optimization systems with numerical variables, but it also gives a qualitative description of the system that is understandable and can be easily understood.

### 3. APPLICATIONS OF FUZZY LOGIC IN MEDICINE

Fuzzy logic permits the inclusion of human heuristics into computer-assisted decision-making that may be applied to individual patients. This is possible because fuzzy logic takes into consideration all of the nuances and complexity that are associated with people. The major objective of this chapter is to provide a comprehensive assessment of the many applications of fuzzy logic that are currently being used in the medical field.

Knowledge in the field of medicine refers to the connection that may be made between symptoms and diagnoses, which a physician could discover from reading books, journals, or monographs, but also through hands-on experience. The second half of the 20th century saw the beginning of the computerization process for the storing of medical information. The idea of fuzzy sets has been used to the creation of medical expert systems, which are designed to provide assistance to doctors in decision-making and in providing care to patients. One definition of fuzzy logic is that it is "a class of objects with a range of membership grades." In contrast to evidence-based medicine, which can only be applied to a certain number of patients at a time, this method takes into consideration all of the intricacies and differences that are unique to each person and provides a statistical analysis that is suitable for a "individual." It provides the scientific community with assistance in analyzing all of the potential outcomes and identifying the degree to which the illness has progressed. In mathematics and engineering, fuzzy logic is a well-established concept; but, until the last decade, medical professionals were

unaware of its potential applications in the field. The number of medical articles that made use of fuzzy logic climbed from two per year in 1991 to one hundred seventy-five per year in 2002, as shown by a recent piece of study. Up until this year, a search for "fuzzy logic" in Medline would have returned more than 1600 papers. A more recent search on Medline, on the other hand, turned up a total of 2448 publications, of which more than 300 were published in the prior calendar year. This illustrates that the scientific community and the medical industry rely heavily on fuzzy logic.

#### ➤ Fuzzy Logic In Medicine

- Sources of Uncertainty

Because of all the complexity involved in medical practice, typical quantitative methodologies are not suitable for data analysis. In the field of medicine, it is not uncommon for there to be a lack of knowledge, for that information to be imprecise, and for it to sometimes even be conflicting. The following categories may be used to organize the many causes of unpredictability.

- **Information about the patient:** It is common practice to inquire about a patient's medical history from the patient, as well as from the patient's family. This is something that is often random and unclear.
- **Physical examination:** The doctor will often collect factual information; yet, it is not always possible to draw a clear boundary between a normal scenario and a

one that may be abnormal.

- **Results:** Results of laboratory and other diagnostic tests are also subject to some mistakes and even to improper behavior of the patient prior to the examination.
- **Symptoms:** The patient may include simulated, exaggerated, understated symptoms, or may even fail to mention some of them.
- **Classification:** We want to draw attention to the disparity between the rise in the prevalence of mental illnesses and the lack of biological categorization. Particularly when using a categorical method of diagnosis, it is challenging to describe critical circumstances, which may also be referred to as borderline illnesses.

➤ **Medical Applications:**

Fuzzy logic plays a key function in medicine. The following are some instances of how fuzzy logic might be used to various types of illness classifications.

- The granulating of information that is unclear in medical pictures.
- Extraction of blood vessels from three-dimensional MRA images.
- Making decisions and monitoring consciousness while the patient is under general anesthesia.
- Sifting through medical data in search of fuzzy association rules.
- Calculating an individual's likelihood of developing coronary heart disease by using a decision-making process that makes use

of fuzzy logic.

- A temporal abductive diagnostic paradigm that was conceived of based on a model of a cardiac intensive care unit.
- A Fuzzy Model for Recognizing Patient Evolutionary Patterns
- To forecast how patients suffering from alcoholism would respond to the treatment with citalopram.
- Conduct diabetic neuropathy tests and check for diabetic retinopathy as soon as possible.
- Using data from magnetic resonance imaging (MRI), determine the volume of the brain's tissue and evaluate the results of functional MRI.
- To assist in the diagnosis of astrocytic tumors that are seen in the central nervous system.
- Differentiating between benign and malignant melanomas
- To refine the method by which decisions on radiation treatment are made.
- To see the functioning of nerve cells inside the brain.
- To offer quantifiable data about the intake of drugs.
- For the early diagnosis of breast cancer, prostate cancer, or lung cancer.
- Specify the different forms of strokes, as well as the simultaneous ischemic stroke.
- To maintain a healthy blood pressure level while receiving anesthesia.
- Identifying flexor-tendon rehabilitation procedures.
- In order to establish the correct amount of lithium to take.
- To investigate the auditory P50 subtype of the schizophrenia disorder.

The examination of fuzzy epidemics, the making of nursing decisions, and the overcoming of adaption to electro acupuncture are only a few of the many additional possible applications. The



process of diagnosing illnesses is inherently riddled with ambiguity and imprecision due to the nature of medicine. When it comes to symptoms, the same illness might present itself in a variety of unique ways based on the individual patient as well as the degree to which they are affected. There are a variety of conditions that might manifest with the same symptom. On the other hand, the patient's several conditions can interact in a way that makes it difficult to define any one of them in the conventional sense. When it comes to disease entities, the greatest and most complete explanations use linguistic terms that are inaccurate and unclear. In addition, the traditional conceptions of health and sickness are polar opposites of one another and are incompatible with one another. On the other hand, some more contemporary ways of thinking consider both of these concepts to be complimentary processes that operate along the same continuum. In its official definition, "health" is described as "a state of complete physical, mental, and social well-being and not merely the absence of disease or infirmity" by the World Health Organization (WHO). Disease, illness, and sickness are the three conditions that may lead to a decline in one's health. We have something called fuzzy logic at our disposal so that we can cope with ambiguity and imprecision. The use of fuzzy logic results in the introduction of partial truth values that may range anywhere from true to false. According to the principles of Aristotelian logic, a proposition or circumstance may only take on one of two possible logical forms: either it is true or it is false, either it is black or it is white, or it receives a score of 1-0. The majority of the time in actual life, neither black nor white adequately describes the situation at hand. As a direct

result of this, intermediate logical values may be taken into consideration in a wide range of contexts.

Consider a very fundamental medical scenario as a means of illustrating the points being made. Think about the phrase "you are healthy" for a moment. If you just have a broken nail, does it still apply to you? If you have advanced cancer, does that still hold true? Everyone experiences some degree of illness and health to varying degrees. If you are in perfect health,  $h$  will always equal 1, and  $i$  will always equal 0. The vast majority of people struggle with very little health issues, and  $h = 1$ , yet  $h + i = 1$ . On the other hand, if your  $h$  value is 0 and your  $i$  value is 1, it means that you are completely unwell (or dead). You may probably assume  $h = 0.999$ ,  $i = 0.001$  if you have a broken nail. If you have a severe stomach ulcer, you can probably assume  $i = 0.6$ ,  $h = 0.4$ . On the other hand, if you have terminal cancer, you can probably assume  $i = 0.95$ ,  $h = 0.05$ . Because uncertainty is becoming an increasingly essential factor in research, fuzzy logic is a strategy that may be used to express uncertainty and deal with it in a clear manner. Fuzzy logic is a qualitative computational approach. It is possible that fuzzy set theory may be considered a feasible formalism for dealing with the imprecision that is inherent in many biomedical and bioinformatics difficulties. This is due to the fact that uncertainty is prevalent in fields such as medicine. A technique known as fuzzy logic is put to use in the field of medicine to clarify unclear facts.

#### 4. CONCLUSION

A comparison of the traditional technique versus the fuzzy logic method, along with

an illustration of the most effective way to evaluate student performance, is shown here. The data obtained from the computer indicate that there is a linear connection between the conventional method and the fuzzy 1 scenario. When contrasted with the method that came before it, the second set, which is referred to as fuzzy 2, generates a different sort of value. As a result of the students' performance in this project, the classical technique and fuzzy scenario 1 both provide equal conclusions; however, the significant difference may be assessed using the classical approach and fuzzy scenario 2. The results of the conventional method and the fuzzy scenario 2 are contrasted here to highlight the differences in performance values. However, as compared to the traditional method, fuzzy 2 performs better for score values that are more than 50. For score values that are less than 50, the performance value of fuzzy 2 is lower than the performance value of the traditional approach.

In this paper, we discuss a method based on fuzzy logic that may be used to solve nonlinear optimization problems that also include linear approximation relationships. Using the proposed method, which linearly approximates the nonlinear objective function as well as the constrained functions forming the feasible regions around an initial feasible point by Taylor's Series, the nonlinear optimization problem is then roughly equivalent to a linear optimization problem or a linear programming problem. This is because the proposed method linearly approximates the nonlinear objective function as well as the constrained functions forming the feasible regions around an initial feasible point.

Quantitative analysis, in addition to

qualitative evaluation of medical data, is very necessary if one is interested in gaining an understanding of the disease process in its earliest phases. Early diagnosis may be significantly aided by the use of several imaging methods in medical practice. Enhancement of conventional diagnostic procedures for medical imaging may be possible with the use of fuzzy pattern recognition algorithms. Researchers have presented a variety of algorithmic approaches for fuzzy pattern recognition in an effort to enhance the creation of new processes as well as medical diagnosis and prognosis. This is being done in an effort to progress the field of study. There are a variety of challenges that are effectively addressed by the various approaches. It is challenging to do research in the field of fuzzy pattern recognition from both a theoretical and an applied perspective. It has use in medical diagnostics, particularly for identifying early signs of illness.

The extent to which a person's health may be considered an essential measure of human development is directly proportional to that person's potential to lead a life that is both socially and economically useful. This chapter examines the use of multiplication and addition in Sanchez's medical diagnostic methodology by applying two distinct algorithms to the same case. The goal of this investigation is to better understand how Sanchez's method works. According to the results of a numerical calculation, there is no discernible difference between the two approaches. Calculations involving addition, on the other hand, are simpler to carry out than those involving multiplication. By using the technique of multiplication, the connection matrix may be comprehended with relative ease. It

presents a clear picture of fuzzy logic as a method that is cautious and focused when applied to a wide range of therapeutic issues. It provides an explanation of the fundamental ideas and methods that underlie the application of fuzzy set theory to health-related issues that occur in the real world. Increased generality, improved modeling of real-world challenges, and greater capacity to express oneself more fully in specialized subfields of non-fuzzy mathematics are some of the benefits that may be gained via the use of fuzzy mathematics.

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From the first page: “If the base 2 is used the resulting units may be called binary digits, or more briefly, bits, a word suggested by J. W. Tukey.”. This is the first known printed instance of the word ‘bit’ with the meaning of binary digit.

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