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Dynamics of a Homogeneous

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Abstract:

The possible ways of dynamics of a homogeneous and isotropic space described by the Friedmann–Lemaitre–Robertson–Walker metric in the framework of cubic in the Ricci scalar f(R) gravity in the absence of matter are considered. This paper points towards an effective method for limiting the parameters of extended gravity models. A method for f(R)-gravity models, based on the metric dynamics of various model parameters in the simplest example is proposed. The influence of the parameters and initial conditions on further dynamics are discussed. The parameters can be limited by (i) slow growth of space, (ii) instability and (iii) divergence with the inflationary scenario.

Keywords: *modified gravity; f(R) gravity; cosmology*

1. INTRODUCTION

In spite of the achievement of exploratory trial of the hypothesis of general relativity (GR) with superb exactness [1–3], the investigation of different adjustments of the hypothesis of gravity keeps on dating. Truly, the principal endeavors at changing GR were pointed towards the unification of gravity with different collaborations by adding higher measurements [4,5]. Current interest in altered gravity has expanded with the development of a huge arrangement of observational cosmology information [6]. The quick advancement of exploratory cosmology has given occasion to feel qualms about the Big Bang hypothesis. The standard cosmology of the Big Bang was depicted in the structure of GR. On account of a homogeneous and isotropic space, Einstein's conditions lead to the Friedmann arrangements [7], which depict the phases of strength of radiation and matter. Nonetheless, present day observational information demonstrate the presence of phases of sped up extension of the universe. The first is the swelling theory, which isn't simply needed to take care of levelness and skyline issues, yet in addition to clarify the almost level temperature anisotropy range saw in the infinite microwave foundation [8]. The second is the advanced sped up extension stage [9,10]. These two phases of the sped up extension of the universe can not be clarified as far as standard matter with the known condition of state in the structure of GR. Notwithstanding, these marvels can be clarified in the structure of adjusted gravity.

Probably the least difficult way to deal with changed gravity is f(R) gravity, with f(R) being capacity of the Ricci scalar R. This class of speculations is generally utilized in current exploration [11–14] and, at times, effectively tackles specific issues and fits the observational cosmology information [15–18]. The first and best plan having a place with the f(R) class of speculations was the Starobinsky model [19] containing just a single free boundary. In this model, the expansion of the R 2–term was made for end of cosmological peculiarity and prompted the inflationary stage. The Starobinsky's inflationary model is a specific answer for the class of hypotheses of gravity with higher subsidiaries, which are without phantom levels of opportunity, perturbatively unitary and limited at the quantum level [20]. This model has a "agile exit" from expansion and gives a system to the resulting creation and last thermalization of the standard matter. Notwithstanding, adding a cubic term may give a superior arrangement inflationary information, as was as of late displayed in [21] In this paper, a method of considering the conceivable methods of development of a homogeneous and isotropic space in the structure of cubic f(R) gravity is proposed. Thinking about just the gravitational part of the development, the impact of boundaries and beginning conditions on additional elements are examined

2. Basic Equations

Let us consider the theory with the following action:

$$S[g_{\mu\nu}] = \frac{m_{\text{Pl}}^2}{2} \int d^4x \sqrt{|g|}\, f(R)\,. \tag{1}$$

The rationalized Planck units h¯ = c = kB = 8πG = 1 are used where h¯ is the reduced Planck constant, kB is the Boltzmann constant, c is the speed of light, and G is the Newtonian gravitational constant. Hence, the Planck mass mPl = √ hc¯ /8πG = 1. gµν is the metric tensor with the metric signature (+, −, −, −). The indices denoted by Greek letters take on the values 0, 1, 2, 3.

$$f_R(R)R_{\mu\nu} - \frac{1}{2} f(R)g_{\mu\nu} + \left(\nabla_\mu \nabla_\nu - g_{\mu\nu}\Box\right) f_R(R) = 0\,. \tag{2}$$

Here, $\partial_\mu \equiv d/dx^\mu$, $\Box \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$ and $f_R(R) \equiv df(R)/dR$.

Considering the metrics of homogeneous and isotropic spaces, namely,

$$ds_+^2 = dt^2 - e^{2\alpha(t)}\left(dx^2 + \sin^2 x\, dy^2 + \sin^2 x\, \sin^2 y\, dz^2\right), \tag{3}$$

$$ds_0^2 = dt^2 - e^{2\alpha(t)}\left(dx^2 + dy^2 + dz^2\right), \tag{4}$$

$$ds_-^2 = dt^2 - e^{2\alpha(t)}\left(dx^2 + \sinh^2 x\, dy^2 + \sinh^2 x\, \sin^2 y\, dz^2\right), \tag{5}$$

corresponding to spaces with positive (3), zero (4) and negative (5) curvatures, from Equations (2), one obtains the nontrivial following equations

$$6\dot{\alpha}\dot{R} f_{RR}(R) - 6\left(\ddot{\alpha} + \dot{\alpha}^2\right) f_R(R) + f(R) = 0\,, \tag{6}$$

$$2\dot{R}^2 f_{RRR}(R) + 2\left(\ddot{R} + 2\dot{\alpha}\dot{R}\right) f_{RR}(R) - \left(2\ddot{\alpha} + 6\dot{\alpha}^2 + 4\gamma e^{-2\alpha(t)}\right) f_R(R) + f(R) = 0\,, \tag{7}$$

where the Ricci scalar for the metrics used is

$$R(t) = 12\dot{\alpha}^2(t) + 6\ddot{\alpha}(t) + 6\,\gamma e^{-2\alpha(t)} \tag{8}$$

with γ = +1 for metric (3), γ = 0 for metric (4) and γ = −1 for metric (5). The dot denotes the time derivative d/dt

In order to find a solution, a system to be determined, consisting of Equation (7) and the definition the Ricci scalar, Equation (8). Equation (6) to be used as a constraint on the initial condition. Let us choose the initial conditions for the unknown functions α(t) and R(t) of the system of Equations (7) and (8) as

$$\alpha(0) = \alpha_0, \quad \dot{\alpha}(0) = \alpha_1, \quad \dot{R}(0) = R_1 \tag{9}$$

and then the initial value of the curvature R(0) = R0 to be obtained by solving Equation (6).

The choice of the initial conditions is the most difficult task. In the formulation of the problem used here, the conditions (9) are free parameters that one introduces"by hand"

In addition to the problem of choosing the initial conditions, some models contain the possibility of several asymptotic values of the Ricci scalar for a particular form of the f(R) function. Here, the simplest case when the chosen function has the form,

$$f(R) = a_3 R^3 + a_2 R^2 + R, \tag{10}$$

is considred. In this case, the trace of Equations (2) at the constant curvature, Rc = const, leads to

$$f_R(R_c) R_c - 2 f(R_c) = 0, \tag{11}$$

which defines the asymptotic values of the Ricci scalar. For the simplest case, when more than one asymptotic value is possible, from Equation (11), one obtains:

$$R_c = 0, \; R_c = \frac{1}{\sqrt{a_3}}, \text{ and } R_c = -\frac{1}{\sqrt{a_3}}. \tag{12}$$

So what conditions lead to the realization of a particular asymptotics?

3. Analysis in the Einstein Frame

In this Section, the effect of the model parameter values and initial conditions on the dynamics of the spaces are verified. The action (1) can be reduced to scalar-tensor (ST) theory by introducing an auxiliary scalar field χ as a result of the Legendre transformation [22]:

$$S_{\mathrm{ST}} = \frac{1}{2}\int d^4x\sqrt{-g}\left[f(\chi) + f'(\chi)(R-\chi)\right], \tag{13}$$

where f 0 (χ) ≡ d f /dχ. Upon conformal transformation, gμν = | f 0 (χ)| −1 gˆμν one obtains this action in the Einstein (E) frame:

$$S_{\mathrm{E}} = \frac{1}{2}\int d^4x\sqrt{-\hat{g}}\left[\hat{R} + \hat{g}^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - 2V(\psi)\right], \tag{14}$$

$$\psi = \sqrt{\frac{3}{2}}\ln f'(\chi), \quad V(\psi) = \left.\frac{\left(\chi f'(\chi) - f(\chi)\right)}{2\left(f'(\chi)\right)^2}\right|_{\chi=\chi(\psi)}. \tag{15}$$

To avoid the antigravity regime, the condition f 0 (χ) > 0 [23], is set. One of the classical equations of the action (13) is f 00(χ) R − χ = 0 and, then, R = χ if f 00(χ) 6= 0. Using the form (15) of the potential V(ψ), a condition

$$\frac{dV(\psi)}{d\chi} = -\left.\frac{f''(\chi)\left(\chi f'(\chi) - 2f(\chi)\right)}{2\left(f'(\chi)\right)^3}\right|_{\chi=\chi(\psi)} \xrightarrow{f''(\chi)\neq 0} \chi f'(\chi) - 2f(\chi) = 0, \tag{16}$$

for a local extremum of an auxiliary scalar field can be found. The condition (16) is exactly the condition (11) obtained above, which leads to a de Sitter space endowed with constant curvature. The existence of a stable minimum for a scalar field then follows f

$$\frac{d^2V(\psi)}{d\chi^2} = \left.\frac{f''(\chi)\left(-\chi f''(\chi) + f'(\chi)\right)}{2\left(f'(\chi)\right)^3}\right|_{\chi=\chi(\psi)} > 0 \to -\chi + \frac{f'(\chi)}{f''(\chi)} > 0. \tag{17}$$

From the condition (17) one gets for chosen form

$$-R_c + \frac{3a_3R_c^2 + 2a_2R_c + 1}{6a_3R_c + 2a_2} = \begin{cases} \frac{1}{2a_2}, & \text{for } R_c = 0, \\ -\frac{1}{a_2 \pm 3\sqrt{a_3}}, & \text{for } R_c = \pm\frac{1}{\sqrt{a_3}}. \end{cases} \quad (18)$$

One of the asymptotics compares to the limit of the potential, while the other one relates to its base contingent upon the upsides of the boundaries of the capacity (10). The potential is displayed in Figure 1 for various indications of the boundaries of the capacity (10). It ought to be noticed that the presented worth of ψ is identified with the assistant scalar field χ by Equation (15) with χ = R. An unsteady position is conceivable everywhere ψ values in the extreme left and extreme right plots of Figure 1, where the two coefficients a3 and a2 have a similar sign. The underlying conditions in circumstance at the extreme left plot can assume a significant part prompting a temperamental arrangement. The picked introductory conditions (9) and upsides of the coefficients should prompt a worth R0 < Rc = 1/√ a3 to guarantee the steadiness of the arrangement. Shakiness in the extreme right plot emerges paying little heed to the decision of beginning conditions. The elements for the other two instances of the boundaries esteems, as displayed in the second and third plots of Figure 1, are very unsurprising. Notwithstanding the underlying worth of the shape, the arrangements will tend to and afterward arrive at a steady least.

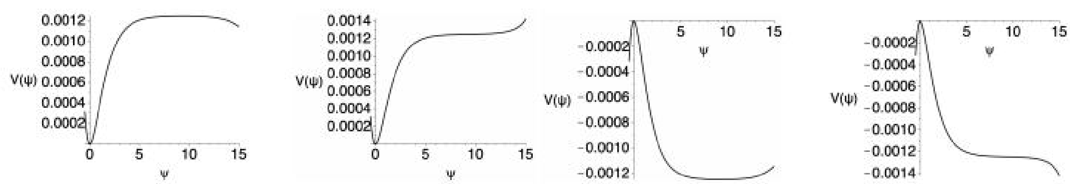

Figure 1. The potential $V(\psi)$ (15) for different values of the parameters of the function (10): $a_3 = 0.01$, $a_2 = 100$ (first); $a_3 = -0.01$, $a_2 = 100$ (second); $a_3 = 0.01$, $a_2 = -100$ (third) and $a_3 = -0.01$, $a_2 = -100$ (last).

Thus, the influence of the values of the parameters of the f(R) function (10) is revealed determining the implementation of the asymptotics (12). The only exception in the model (10) is the case of the far left picture, where the initial conditions can lead to unstable solutions. The obtained statements be confirmed by numerical calculations in the what follows.

4. Numerical Results

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Let us illustrate what was discussed above with the example of a numerical solution for a flat space (4). As it is said in Section 2, a system of Equations (7) and (8) to be solved under the initial conditions (9), chosen near the sub-Planck scale,

$$\alpha_0 \sim -\ln H_{\text{sub-Planck}}\,, \quad \alpha_1 \sim H_{\text{sub-Planck}}\,, \quad R_1 = 0\,, \tag{19}$$

where Hsub-Planck . mPl, and the initial value of the curvature R0 to be found by solving Condition (6). The aftereffects of the mathematical answer for this Cauchy-type issue are displayed in Figures 2 and 3 utilizing the excused Planck units. Figure 2 relates to the instance of the extreme left plot of Figure 1, and Figure 3 compares to the third plot of Figure 1. In Figure 2, the mathematical arrangement covers a few phases of the development of the universe. The first is the inflationary stage, where space develops dramatically and the ebb and flow of room diminishes. Then, at that point, having arrived at nothing, the shape begins to waver around nothing and the Hubble boundary, i.e., α˙(t), at this stage, asymptotically arrives at nothing, and the size of the space, α(t), keeps an eye on a steady. After the damping of the motions of the relating amounts, a temporary system starts to the GR. Notwithstanding, the space doesn't extend enough to portray the apparent piece of the universe. The worth should be α(t∞) > 140, where e 140m −1 Pl relates to the skyline scale ∼ 1028 cm right now. Notwithstanding the last size of room, one can see that the outstanding development of room closes sooner than the comparing term of the inflationary stage. The remarkable extension of room should keep during δt ∼ 107 (in the supported Planck units). During this time, the capacity α(t) should change by the worth Δα(δt) = Ne ≈ 60.

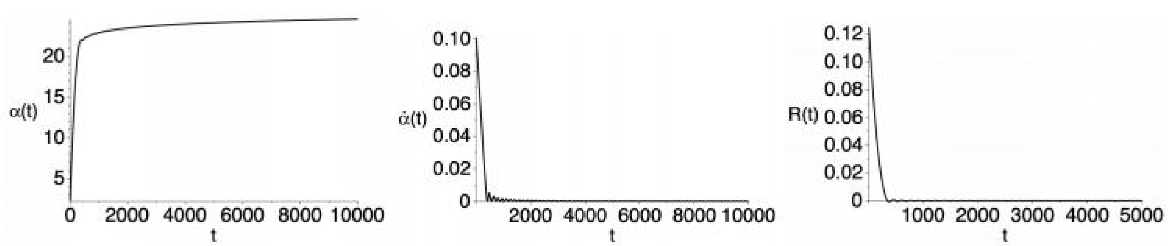

Figure 2. The solution of the system of Equations (7) and (8) with parameters $a_3 = 0.01$, $a_2 = 100$ with the initial conditions $\alpha_0 = 2.3$, $\alpha_1 = 0.1$, $R_0 = 0.125$ and $R_1 = 0$. The asymptotic behavior is $R_c = 0$ and the Hubble parameter $H \equiv \dot{\alpha}(t_\infty) = 0$. The parameter values a_3 and a_2 correspond to the leftmost plot in Figure 1. The rationalized Planck units $\hbar = c = k_B = 8\pi G = 1$ are used, so the Planck mass, $m_{\text{Pl}} = 1$.

5. conclusion

Thinking about just the gravitational elements of a homogeneous and isotropic space, permits us to come to limitations on f(R) work. The asymptotics if there should arise an occurrence of cubic f(R) work are completely controlled by the signs and upsides of the coefficients. By and by, the decision of the underlying conditions can influence the dependability of the arrangement. Notwithstanding the right asymptotic worth of the bend, it is additionally worth thinking about the conceivable size of room, no not exactly the size of the noticeable piece of the universe and the upsides of cosmological boundaries accessible from the observational information [6]. A point by point investigation of cubic gravity in the inflationary situation was as of late acted in [21]. It was shown that an expansion of Starobinsky's model considers better concurrence with trial information. Notwithstanding, considering the simply gravitational elements of an isotropic and homogeneous space [24] the adequate scope of upsides of the coefficients of f(R) work has a convergence with the scope of qualities from [21], not exactly every one of them gives independently. In addition, at Planck energy scales, changes can prompt the development of lopsided spaces. The depiction of this cycle should be possible simply by the hypothesis of quantum gravity, which has not yet been created. The spaces of various arches have various paces of development because of our mathematical arrangements of the traditional conditions of movement. Then, at that point, can our Universe be homogeneous and have isotropic space outside the apparent part? This issue will be examined in future investigations. Subsidizing: This exploration got no outside financing

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