INTERNATIONAL JOURNAL OF TECHNO-ENGINEERING
IJTE

ISSN: 2057-5688

Some Concepts in Numerical Method for Solving Non-Linear Equation

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Abstract- This paper will focus on the numerical methods involved in solving systems of nonlinear equations. First, we will study Powell's method for finding a local minimum of a function, where the function must be a real-valued function of a fixed number of Real-valued inputs. Second, we will examine Regular falsi method which is also called False-position method this method has been described as a generalization of the Bisection Method from which we can try for a better convergence, at the risk of a worse one, or none at all. And third, to calculate the higher order Lagrange's interpolation of the given function, we will study Neville's method we will also give working examples and application of Powell's method, Regular-Falsi method and Neville's method. Which are playing very important role in engineering and science and technology.

1. INTRODUCTION

Over the years, we have been taught on how to solve equations using various algebraic methods. These methods include the substitution method and the elimination method. Other algebraic methods that can be executed include the quadratic formula and factorization. In Linear Algebra, we learned that solving systems of linear and non-linear equations can be implemented by using row reduction as an algorithm. However, when these methods are not successful, we use the concept of numerical methods. Numerical Methods are used to approximate solutions of equations when exact solutions cannot be determined via algebraic methods. They construct successive approximations that converge to the exact solution of an equation or system of equations. In this paper we focused on solving nonlinear equations involving with various variables. We used methods such as Powell's method, the Regular-Falsi method, and the

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Neville's method. Problems are only focused on solving nonlinear equations with different variable, rather than nonlinear equations with only one variables. The goal of this paper is to examine three different numerical methods that are used to solve systems of nonlinear equations in several variables. These method we will look at is Powell's method. This will be followed by Regular-Falsi method, which is sometimes called a False-position method. Lastly, we will study the Neville's method that is used to calculate higher order Lagrange's interpolation polynomial of nonlinear equations. For each method, a breakdown of each numerical procedure will be provided. In addition, there will be some discussion of the convergence of the numerical methods, as well as the applications of each method. After a discussion of each of the three methods, we will be given working examples for each method and its application.

2. BASIC DEFINITIONS:

A real valued function f defined on a domain x has an absolute maximum point at * x if * f x f x ( ) ( ) ≥ for all x in X. similarly the function has an absolute minimum point at * x if * f x f x ( ) ( ) ≤ for all x in X. the value of the function at maximum point is called the maximum value of the function and the value of the function at a minimum point is called the minimum value of the function. Note: The maxima and minima of a function known collectively as extrema. 2) GOLDEN RATIO The Golden ratio has unique mathematical properties. It is the only positive number whose square is one greater than itself. it is the only positive number whose reciprocal is one less than itself. It is also the found in limits and Fibonacci series Its symbol is phi (ϕ) it is the special number approximately equals to 1.618 3) DIRECTION SET METHOD Powell's method is also known as Direction set method in which by using linmin we move along the first direction to its minimum then from there along the second direction to its minimum and so on cycling through the whole set of directions as many times as necessary. This simple method is actually not too bad for many functions. 4) CONJUGATE DIRECTION Having performed a line minimization along a direction we would like to choose a new direction V so that minimizing along V will not spoil the minimization along U. we can determine such a direction by using the Taylor approximation at

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$$f(a+x) \approx c - bx + \frac{1}{2} x^T Ax$$

If we have just minimized along a direction u then the component of the gradient along u must be zero, thus the gradient itself is perpendicular to u

5) BRACKETING MINIMUM A minimum is bracketed by three points, if and is less than both and if this condition holds and the function continuous in the interval, than has a minimum for some, a very simple iterative minimum bracketing procedure follows start from any initial point. 6) BISECTION METHOD The bisection method in mathematics is a root finding method that repeatedly bisects an interval and then select a subinterval in which a root must lie for further processing. It is very simple and robust method but it is also relatively slow. 7) SIMPLE BRACKETING METHOD This is one of the simplest and reliable iterative method for the solution of non linear equation, this method is also known as binary chopping or half interval method. 8) UNDER INTERPOLATION Interpolation is an estimation of a value within two known values in a sequence of values polynomial interpolation is a method of estimating value between known data points. 9) SOR METHOD: In numerical linear algebra the method of successive over relaxation is a variant of the Gauss-seidel method for solving a linear system of equations. Resulting in faster convergence. 10) DIVIDED DIFFERENCE Divided difference is an algorithm historically used for computing tables of logarithmic and trigonometric functions. Divided difference is a method can be used to calculate the coefficient in the interpolation polynomial in the Newton form. 11) LAGRANGE POLYNOMIAL Lagrange polynomial are used for polynomial interpolation for a given set of points with no two value3s equal to the Lagrange polynomial is a polynomial of lowest degree that assumes each values and the corresponding values. 12) ZERO-ORDER APPROXIMATION Zero-order approximation process a chemical reaction in which the rate of change of concentration is independent of the concentration. Zero-order approximation is an approximation of function by a constant. 13) LINEAR APPROXIMATION Linear approximation is the process of finding the equation of a line that is the closest estimate of a function for a given value of x. linear approximation is also

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known as tangent linear approximation and it is used to simplify the formulas associated with trigonometric functions. 14) QUADRATIC APPROXIMATION Quadratic approximation also uses the point to approximate nearby values but uses a parabola instead of just a tangent line to do so

Powell's method strictly Powell's conjugate direction method, is an algorithm proposed by Michael J. D. Powell's for finding a local minimum of a function. The function need not be differentiable, and no derivatives are taken. The function must be real valued function of a fixed number of real valued inputs. The caller passes in the initial point. The caller also passes in a set of initial search vectors. Typically N search vectors (say {s1. . . sn}) are passed in which are simply the normal aligned to each axis. The method minimises the function by a bi-directional search along each search vector, in turn. The bi-directional line search along each search vector can be done by golden section search or Brent's method. The method is useful for calculating the local minimum of a continuous but complex function, especially one without an underlying mathematical definition, because it is not necessary to take derivatives. The basic algorithm is simple; the complexity is in the linear searches along the search vectors, which can be achieved via Brent's method.

Assume that we know how to minimize a function of one variable. If we start at a point p in N-dimensional space, and proceed from there in some Vector direction, then any function of N variables f p( ) can be minimized along the line n by our one-dimensional methods. Different methods will differ only by how they The next direction n to try The line minimization routine linmin is a black-box sub algorithm, whose definition is

• Obviously what we need is a better set of directions than the i e 's. All direction set methods consist of prescriptions for updating the set of directions as the method proceeds, attempting to come up with a set which either. • Includes some very good directions that will take us far along narrow valleys, or else (more subtly). • Includes some number of "non-interfering" directions with the special property that minimization along one is not "spoiled" by subsequent minimization along another, so that interminable cycling through the set of directions can be

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avoided. CONJUGATE DIRECTION • This concept of “non-interfering” directions, called conjugate directions, can be made mathematically explicit. • Assume f is differentiable. If we minimize f along the direction u , then the gradient must b perpendicular to u at the line minimum • Take some particular point p as the origin of coordinate system then any function f can be approximated by its Taylor series

$$f(x) = f(p) + \sum_i \frac{\partial f}{\partial x_i} x_i + \frac{1}{2} \sum_{i,j} \frac{\partial^2 f}{\partial x_i \partial x_j} x_i x_j + \dots$$

Where $c = f(p), b = -\nabla f\big/_p$, A=Hessian matrix.

• When the relation holds pair wise for all members of a set of vectors, they are said to be a conjugate set. • If you do successive line minimization of a function along a conjugate set of directions, then you don't need to redo any of those directions. • The idea behind a direction set method is to come up with a set of N linearly independent, mutually conjugate directions. • Then, one pass of N line minimizations will put it exactly at the minimum of a quadratic form. For functions f that are not exactly quadratic forms, it won't be exactly at the minimum; but repeated cycles of N line minimizations will in due course converge quadratic ally to the minimum.

Powell (1964) showed that, for a quadratic form, k iterations of the above basic procedure produces a set of directions i u whose last k members are mutually conjugate? Therefore, N iterations of the basic procedure, amounting to N N(+1) line minimizations In all will exactly minimize a quadratic form [Brent, 1973] gives proofs of these statements in accessible form. Unfortunately, there is a problem with Powell's algorithm. The procedure of throwing away, at each stage, n u in favor of N 0 p p − tends to produce sets of directions that “folds up on each other" and become linearly dependent. Once this happens, then the procedure finds the minimum of the function f only over a subspace of the full N -dimensional case; in other words, it gives the wrong answer.

INTERNATIONAL JOURNAL OF TECHNO-ENGINEERING
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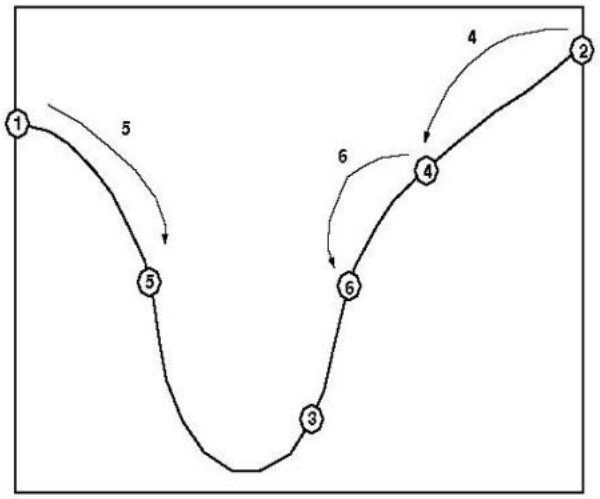
There are a number of ways to x up the problem of linear dependence in Powell's algorithm, among them: We can reinitialize the set of directions i u to the basis vector after every N or N +1 iterations of the basic procedure. This produces a serviceable method, which we commend if quadratic convergence is important for your application (i.e., if your functions are close to quadratic forms and if you Desire high accuracy). We can give up the property of quadratic convergence in favor of a more heuristic scheme (due to Powell) which tries to find a few good directions along narrow valleys instead of N necessarily conjugate directions. One-dimensional Minimization: Golden Section Search



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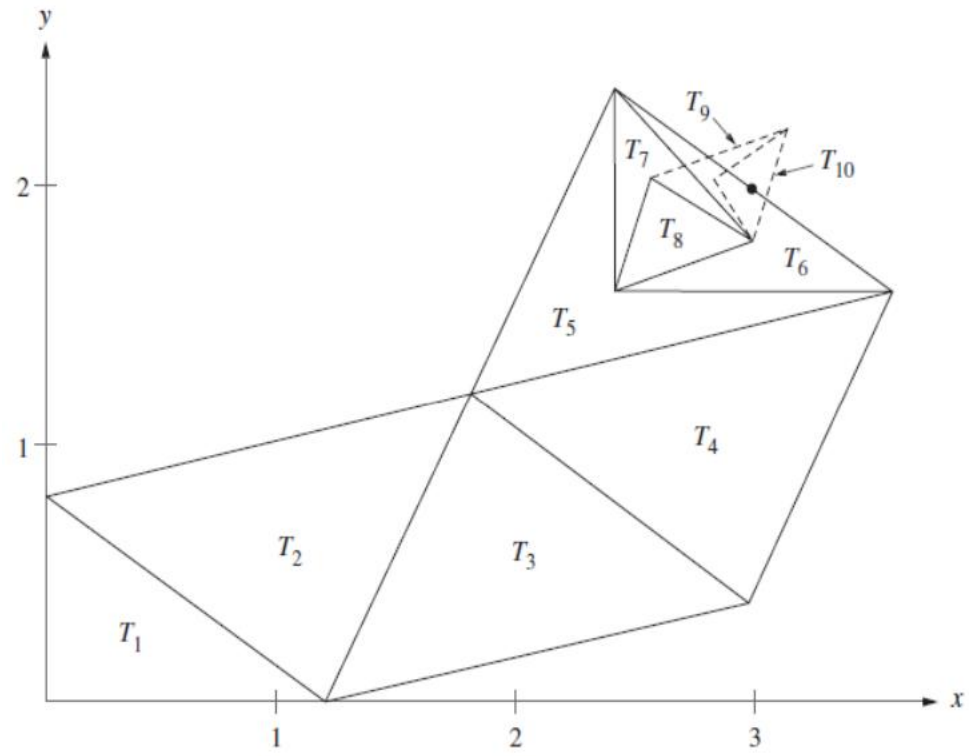

Fig.1 The sequence of triangles {Tk} converging to the point

Points X0 =P0, P1, P2 . . . PN = X1. Along each standard base vector the function f is a function of one variable. Thus the minimization of f requires the application of either the golden ratio or Fibonacci searches on an interval over which the function is unimodal. The iteration is then repeated to generate a sequence of points {Xk }∞ k=0. Unfortunately, the method is, in general, inefficient due to the geometry of multivariable functions. But the step from the point X0 to the point X1 is the first step of Powell's method. The essence of Powell's method is to add two steps to the process described in the preceding paragraph. The vector PN − P0 represents, in some sense, the average direction moved during each iteration. Thus the point X1 is determined to be the point at which the minimum of the function f occurs along the vector PN −P0. As before, f is a function of one variable along this vector and the minimization requires an application of the golden ratio or Fibonacci searches. Finally, since the vector PN − P0 was such a good direction, it replaces one of the direction vectors for the next iteration. The iteration is then repeated using

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the new set of direction vectors to generate a sequence of points {Xk }∞ k=0. The process is outlined below.

EXAMPLE

Use the process described in the preceding paragraph to find X1 and X2 for the function f x y x y (, cos sin .) = + ( ) ( ) Use the initial point 0 X = (5.5,2).

SOLUTION

Let U=$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $P_0 = X_0 = (5.5, 2)$. when $i = 1$

The function,

$$f(P_0 + \gamma_1 U_1) = f((5.5,2) + \gamma_1(1,0))$$

$$= f(5.5 + \gamma_1, 2)$$

$$= \cos(5.5 + \gamma_1) + \sin(2)$$

Has a minimum at $\gamma_1 = -2.3584042$. Thus $P_1 = (3.1415958, 2)$. When $i = 2$

The function

$$f(P_1 + \gamma_2 U_2) = f((3.1415958, 2) + \gamma_2(0, 1))$$

$$= f(3.1415982, 2 + \gamma_2)$$

$$= \cos(3.1415982) + \sin(2 + \gamma_2)$$

Has a minimum at $\gamma_2 = 2.7123803$. Thus $P_2 = (3.1415958, 4.7123803)$.

Set $U_2' = (P_2 - P_0)'$

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Set $U_2' = (P_2 - P_0)'$

$$U = \begin{bmatrix} 0 & -2.3584042 \\ 1 & 2.7123803 \end{bmatrix}$$

The function

$f(P_0 + \gamma U_2) = f((5.5, 2) + \gamma(-2.3584042, 2.7123803))$

$= f(5.5 - 2.3584042\gamma, 2 + 2.7123903\gamma)$
$= \cos(5.5 - 2.3584042\gamma) + \sin(2 + 2.7123803\gamma)$

Has a minimum at $\gamma = 0.9816697$. Thus $X_1 = (3.1848261, 4.6626615)$.

Set $P_0 = X_1$. When i = 1 the function

$f(P_0 + \gamma_1 U_1) = f((3.1848261, 4.6626615) + \gamma_1(0, 1))$

$= f(3.1848261, 4.6626615 + \gamma_1)$
$= \cos(3.1848261) + \sin(4.6626615 + \gamma_1)$

Has a minimum at $\gamma_1 = 0.0497117$. Thus $P_1 = (3.1848261, 4.7123732)$.

When i = 2 the function

$f(P_1 + \gamma_2 U_2) = f((3.1848261, 4.7123732) + \gamma_2(-2.3584042, 2.7123809))$

$= f(3.1848261 - 2.3584042\gamma_2, 4.7123732 + 2.7123809\gamma_2)$
$= \cos(3.1848261 - 2.3584042\gamma_2) + \sin(4.7123732 + 2.7123809\gamma_2)$

Has a minimum at $\gamma_2 = 0.0078820$. Thus $P_2 = (3.1662373, 4.7337521)$.

Set $U_2' = (P_2 - P_0)'$ and

$$U = \begin{bmatrix} -2.3584042 & -0.0185889 \\ 2.7123803 & 0.0710906 \end{bmatrix}$$

The function

$f(P_0 + \gamma U_2) = f((3.1848261, 4.6626615) + \gamma(-0.0185889, 0.0710906))$

$= f(3.1848261 - 0.0185889\gamma, 4.6626615 + 0.0710906\gamma)$
$= \cos(3.1848261 - 0.0185889\gamma) + \sin(4.6626615 + 0.0710906\gamma)$

Has a minimum at $\gamma = 0.8035684$. Thus X2 = (3.1698887, 4.7197876).

The function f (x, y) = cos(x) + sin(y) has a relative minimum at the point

P = (π, 3π/2). The graph of f is shown in Figure2. Figure3. Shows a contour plot of the function f and the relative positions of the points X0, X1, and X2.

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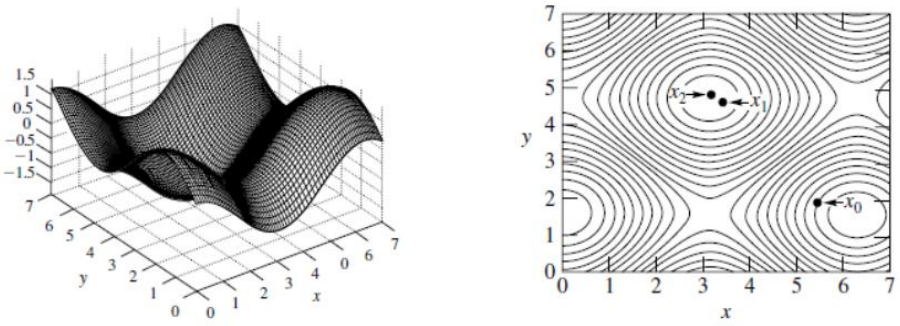

In step (IV) of the previous process the first vector U1 was discarded and the average direction vector PN − P0 was added to the list of direction vectors. In fact, it would be better to discard the vector Ur along which the greatest decrease in f occurred. It seems reasonable that the vector Ur is a large component of the average direction vector UN = PN − P0. Thus, as the number of iterations increase, the set of direction vectors will tend to become linearly dependent. When the set becomes linearly dependent one or more of the directions will be lost and it is likely that the set of points {X}∞ k=0 will not converge to the point at which the local minimum occurs. Furthermore, in step (iv) it was assumed that the average direction vector represented a good direction in which to continue the search. But that may not be the case.

3. APPLICATIONS

1) Powell's method can be applied to the optimization of a flow injection system configuration.
2) The performance of this method has been compared with the modified simplex method the system studied is the determination of ammonia, based on indophenols blue reaction.
3) Powell's method has been used for the evaluation of the surge arrester models parameters. the proper modelling of metal oxides surge arresters and the right selection of equivalent circuit parameters are very significant issues, since quality and reliability of lightning performance studies can be improved with the more efficient representation of the arresters dynamic behaviour.

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4. CONCLUSION

Hence the Powell's method an algorithm proposed by Michael J. D. Powell for finding a local minimum of a function. The function need not be Differentiable and no derivatives are taken by using this method we can calculate the local minimum of a continuous but complex function, especially one without an underlying mathematical definition, because it is not necessary to take derivative.

REFERENCES

1. Powell M.J.D. (1964) "An efficient method for finding the minimum of a function of several variables without calculating derivatives". Computer Journal 7(2): 155-162.

2. Press, W.H., Teukolsky, S.A., Vetterling, W.T., Flannery, B.P., (2007). "Section10.7 Direction set (Powell's) Methods in Multidimensions". pp.286-289. New York: Cambridge Universitry Press.

3. Mathews, John H. "Module for Powell search method for a minimum". Fullerton Retrieved 16 June 2017. California State University.

4. Brent, Richard P. (1973). "Section 7.3., Powell's algorithm". N.J. Prentice-Hall.

5. Jean-Luc Chabert. (Berlin Springer, 1999), "A History of Algorithms, from the Pebble to the Microchip". pp. 86-91.

6. Joseph Needham (1 January 1959) "Science and civilisation in China and the Sciences of the Heavens and the Earth". pp.147. Cambridge University Press. Volume 3.

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ISSN: 2057-5688