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MILD BALANCED AND EQUALLY BALANCED INTUITIONISTIC FUZZY GRAPHS

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ABSTRACT

In this study, we take a look at a new variation on the Double Layered Fuzzy Graph (DLFG). By adjusting parameters, we are able to transform a DLFG into a BDLFG, or a balanced double-layered fuzzy graph. In this study, we derive and verify a few characteristics of BDLFG. We define dense and sparse subgraphs and talk about the characteristics of moderately balanced IFG and equally balanced intuitionistic fuzzy subgraphs. In this article, we examine Intuitionistic Fuzzy Graphs (IFG) and their "sum" and "union" operations on subgraphs. Curriculum and syllabus development in higher education are highlighted as an example of the use of evenly balanced IF subgraphs.

Keywords: Development, Fuzzy, Graph, intuitionistic, Double

INTRODUCTION

Graph theory is a subfield of combinatorics that has many applications in other areas of study, including as geometry, algebra, number theory, topology, IR, OP, and computer science. The goal of fuzzy models is to bridge the gap between the symbolic models used in expert systems and the more conventional numerical models used in the hard sciences and engineering. Connections between things may be shown in a graph. In order to account for uncertainty in such connections, fuzzy graphs were developed. There are now many different kinds of programs that use graphs as a means of knowledge representation. More and more often, fuzzy graph theory is used to simulate real-time systems where the information intrinsic to the system changes in accuracy. The benefits of using intuitionistic fuzzy sets and graphs include increased productivity, decreased implementation costs, and more precise issue analysis.

Intuitionistic fuzzy graphs were introduced by Krassimir T. Atanassov and the operations on intuitionistic fuzzy graphs were defined by Parvathi and Karunambigai. Balanced intuitionistic fuzzy graphs based on density function were introduced by Karunambigai et al. who analysed the results on direct product, semi strong product and

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strong product of two intuitionistic fuzzy graphs. Fuzzy graph structures were first developed by Dinesh and Ramakrishnan. In this research, we have explored the enhanced effect of intuitionistic fuzzy sets and the specific usage of graph structures by working on intuitionistic fuzzy graph structures, including some of their core ideas and attributes. In this study, we present the idea of a fuzzy graph structured according to intuitionistic fuzzy-graph theory (IFGS). Sampathkumar's 2006 introduction of graph structures, also known as generalized graph structures, is a useful generalization of graphs that facilitates the study of multiple relations and their corresponding edges in a single graph.

The characteristics of mild balanced IFG and equally balanced IF subgraphs are discussed in this work. The characteristics of intense subgraphs, weak subgraphs, and evenly balanced IF subgraphs are explored. Properties of IFG are investigated under the stated "sum" and "union" operations. It has also been shown that an IFG with a few strong edges can only become a strong IFG by becoming unbalanced and very mild.

LITERATURE REVIEW

Selvanayaki. S (2017) This work deduces the size, order, and density of irregular interval-valued fuzzy graphs, and explores the idea of a strong and balanced irregular interval-valued fuzzy graph. In addition, certain elementary statements and theorems are offered.

V. Nivethana (2017) We define dense and sparse subgraphs and talk about the characteristics of moderately balanced IFG and equally balanced intuitionistic fuzzy subgraphs. In this article, we examine Intuitionistic Fuzzy Graphs (IFG) and their "sum" and "union" operations on subgraphs.

Kishore Kumar. P.K (2018) As the generalization of fuzzy graphs, ambiguous graphs have been a popular area of study as of late. In this study, we present dense subgraphs and explore weak balanced ambiguous graphs and equally balanced vague subgraphs based on their densities. Analysis of the union and sum operations on subgraphs of ill-defined graphs. Similar work was done on the isomorphic features of the -complement of ambiguous graph structure (VGS). An engaging example application on gas pipeline vulnerability assessment is shown at the end.

PROPERTIES OFINTUITIONISTIC FUZZY GRAPHS

Definition 1.1. An intuitionistic fuzzy set (IFS) on an universe X is an object of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$$

Where $\mu A(x)(\in [0, 1])$ is called degree of membership of $x \in A$, $\nu A(x)(\in [0, 1])$ is called degree of non-membership of $x \in A$, and μA and νA satisfy the following condition: for all $x \in X$, $\mu A(x) + \nu A(x) \leq 1$.

Definition 1.2. An intuitionistic fuzzy relation $R = (\mu R(x, y), \nu R(x, y))$ in an universe $X \times Y$ $(R(X \rightarrow Y))$ is an intuitionistic fuzzy set of the form

$$R = \{\langle (x,y), \mu_R(x,y), \nu_R(x,y)\rangle \mid (x,y) \in X \times Y\}$$

Where $\mu R: X \times Y \to [0, 1]$ and $\nu R : X \times Y \to [0, 1]$. The intuitionistic fuzzy relation R satisfies $\mu R(x, y) + \nu R(x, y) \leq 1$ for all $x, y \in X$.

Definition 1.3. Let $G* = (U, E1, E2, \ldots, Ek)$ be a graph structure and $\nu, \rho 1, \rho 2, \ldots, \rho k$ be the fuzzy subsets of $U, E1, E2, \ldots, Ek$, respectively such that

$0 \leq \rho i(xy) \leq \nu(x) \wedge \nu(y)$, for all $x, y \in U$ and $i = 1, 2, \ldots, k$.

Then $G = (\nu, \rho 1, \rho 2, \ldots, \rho k)$ is a fuzzy graph structure of $G*$.

Definition 1.4. Let $G*$ be a graph structure and G be a fuzzy graph structure of $G*$. If $xy \in \text{supp}(\rho i)$, then xy is said to be a ρi-edge of G.

Definition 1.5. The strength of a ρi-path $x0x1 \ldots xn$ of a fuzzy graph structure G is Vn j=1 $\rho i(xj-1xj)$ for $i = 1, 2, \ldots, k$.

Example 1.1. Let $G* = (U, E1, E2)$ be a graph structure such that $U = \{a_1, a_2, a_3, a_4\}$, $E_1 = \{a_1a_2, a_2a_3\}$ and $E_2 = \{a_3a_4, a_1a_4\}$. Let A, B1 andB2 be intuitionistic fuzzy subsets of U, E1 and E2, respectively, such that

$A = \{(a_1, 0.5, 0.2),(a_2, 0.7, 0.3),(a_3, 0.4, 0.3),(a_4, 0.7, 0.3)\}$,

$B1 = \{(a_1a_2, 0.5, 0.3),(a_2a_3, 0.4, 0.3)\}$,

and $B2 = \{(a_3a_4, 0.4, 0.3),(a_1a_4, 0.1, 0.2)\}$.

Then $\breve{G}s = (A, B_1, B_2)$ is an IFGS of $G*$ as shown in Fig. 1.

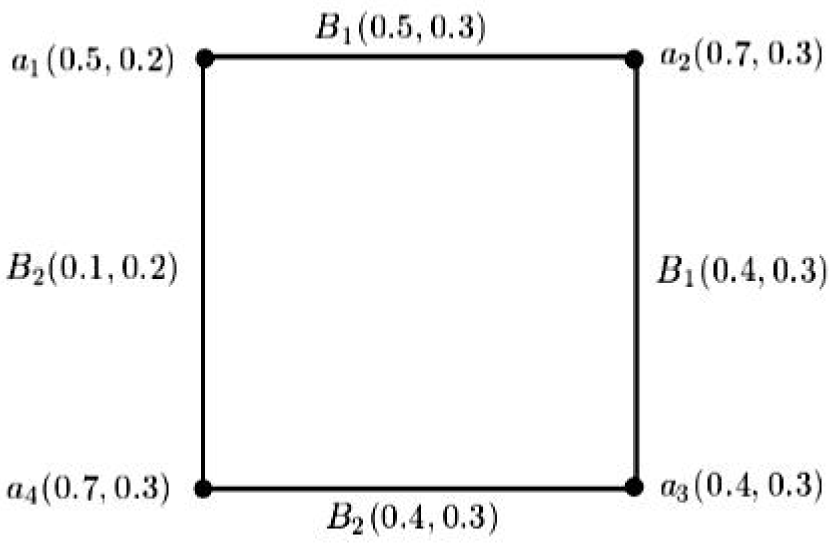

Figure 1. IFGS $\breve{G}(A, B_1,B_2)$

Example 1.2. Consider the IFGS G˘ s = (A, B1, B2), as shown in Fig. 1. Then

(i) a_1a_2, a_2a_3 are B1-edges and a_3a_4, a_1a_4 are B2-edges;

(ii) $a_1a_2a_3$ and $a_3a_4a_1$ are B1- and B2-paths, respectively;

(iii) a_1 and a_3 are B1-connected vertices of U;

(iv) G˘ s is B1-strong, since supp(B1) = $\{a_1a_2, a_2a_3\}$

$\mu B1\ (a_1a_2) = 0.5 = (\mu A(a_1) \wedge \mu A(a_2))$,

$\nu B1\ (a_1a_2) = 0.3 = (\nu A(a_1) \vee \nu A(a_2))$,

$\mu B1\ (a_2a_3) = 0.4 = (\mu A(a_2) \wedge \mu A(a_3))$

and $\nu B1\ (a_2a_3) = 0.3 = (\nu A(a_2) \vee \nu A(a_3))$.

MILD BALANCED INTUITIONISTIC FUZZY GRAPHS

Definition: 2.1 An intuitionistic fuzzy graph (IFG) of the form G: (V, E) said to be a Min-max IFG if

$E \subseteq V\times V$ is finite set of edges such that $\mu B: V\times V\rightarrow[0,1]$ and $\nu B: V\times V\rightarrow[0,1]$ are such that $\mu B(xy) \leq \min\{\mu A(x), \mu A(y)\}$ and $\nu B(xy) \leq \max\{\nu A(x), \nu A(y)\}$ denotes the degree of membership and non-membership of the edges $(x,y)\in E$ and $0 \leq \mu B(xy)+\nu B(xy) \leq 1$ for every $(x, y)\in E$.

Definition: 2.2 In order for H1 = (V1, E1) to be a connected IF subgraph, every pair of vertices in V1 must be reachable by a route.

Definition: 2.3 The term "Intense subgraph" is used to describe a connected subgraph H of an intuitionistic fuzzy graph G: (V, E) if and only if (i). For (ii), V(H) > V(G) and (iii) E(H) > E(G). Both Dv(H) > Dv(G) and D(H) > D(G) are true

Definition: 2.4 A connected subgraph H of an intuitionistic fuzzy graph G: (V, E) is called Feeble subgraph if (i) $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$ and (ii) $D\mu(H) > D\mu(G)$ and $D\nu(H) > D\nu(G)$.

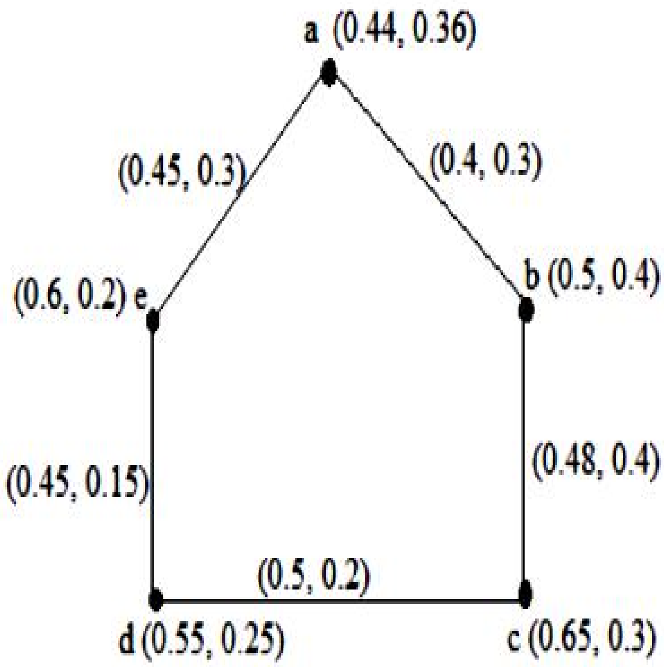

Fig: 1 G:(V,E)

Subgraph	Vertices	Edges	$D_t(H)$	$D_f(H)$
H_1	{a,b}	{ab}	2	2.333
H_2	{b,c}	{bc}	1.333	2.308
H_3	{c,d}	{cd}	1.600	2.154
H_4	{d,a}	{da}	1.800	2.333
H_5	{a,b,c}	{ab, bc}	1.600	2.320
H_6	{b,c,d}	{bc ,cd}	1.455	2.231
H_7	{c,d,a}	{cd , da}	1.689	2.240
H_8	{a, b, c, d}	{ ab, bc, cd}	1.6	2.098
H_9	{b, c, d, a}	{ bc, cd, da}	0.750	2
H_{10}	{c, d, a, b}	{ cd, da, ab}	1.785	2.270
H_{11}	{d, a, b, c}	{ da, ab,bc}	3.086	2.324
H_{12}	{a,b,c,d,a}	{ab, bc, cd,da}	1.853	2.280

Subgraphs of the ill-defined graph G (: V,E) are shown in the above table in terms of their t -density and f -density. The densities of every conceivable linked subgraph of the graph G are listed above. From the data shown above, we may deduce that the subgraphs

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labeled "H3," "H6," "H7," "H8," "H9," "H10," and "H12" are "Intense," "H1," and "H11," are "Feeble," and that "H2," "H4," and "H5" are "Partially Intense and Feeble."

Corollary 2.1. It is impossible for a graph with few strong edges to be somewhat balanced.

Proof: D (H) = 2 t and Df (H) = 2 if and only if the graph G contains some (but not all) strong edges, and if the linked subgraph H has exclusively strong edges. Hence D(H) = (2,2) > D(G). This means the graph cannot be a somewhat balanced ambiguous one.

Example: 2.1 Consider an IFG G:(V,E) with V = {a, b, c, d, e} and E = {ab, bc, cd, de, ea}.

Here are the graph's -density and v-density values:

$$D_\mu(G) = \frac{2(0.4+0.48+0.5+0.45+0.45)}{0.44+0.5+0.55+0.55+0.44} = \frac{4.56}{2.48} = 1.84$$

$$D_\nu(G) = \frac{2(0.3+0.4+0.2+0.15+0.3)}{0.4+0.4+0.3+0.25+0.36} = \frac{2.7}{1.71} = 1.58.$$

Hence, D(G) (1.84,1.58).

BALANCED INTUITIONISTIC FUZZY GRAPHS

First, let's define the density function D(G): D(G)=(D(G), Dv(G)), where G is an intuitionistic fuzzy network defined by (V,E).

Dμ(G) is defined by

$$D_\mu(G) = \frac{2\sum_{u,v\in V}(\mu_2(u,v))}{\sum_{(u,v)\in E}(\mu_1(u)\wedge\mu_1(v))}.$$

and Dv(G) is defined by

$$D_\nu(G) = \frac{2\sum_{u,v\in V}(\nu_2(u,v))}{\sum_{(u,v)\in E}(\nu_1(u)\vee\nu_1(v))}.$$

Definition 3.1 When G = (V,E), a fuzzy graph is intuitively balanced. if D(H) ≤ D(G), that is, Dμ(H) ≤ Dμ(G), Dv(H) ≤ Dv (G) for all subgraphs H of G.

Example 3.1 Consider a IFG, G = (V,E), such that V = {v1, v2, v3, v4}, E = {(v1, v2),(v2, v3), (v3, v4),(v4, v1),(v2, v4)}.

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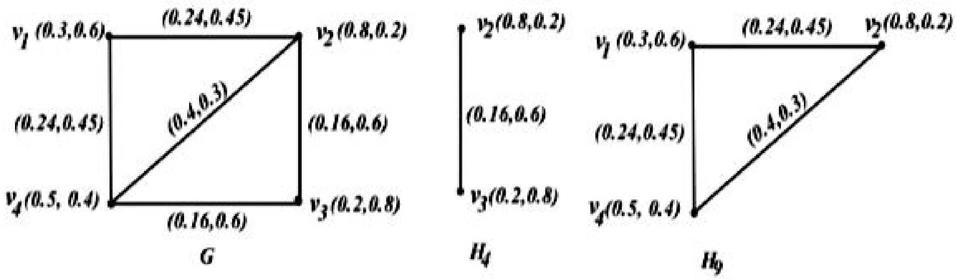

Theorem 3.1: Every complete product IFG is a regular product IFG.

Proof:

Assume G = (V, E) is a full product. IFG As a result, the degree of a vertex's closed neighbors is defined as the sum of the vertex's and its own membership values, while the degree of a vertex's closed g neighbors is defined as the sum of the vertex's and its own nonmembership values. As a result, the degree of closedness between any two vertices will be one. This proves that G is a normal product of the IFG.

Let G = (V, E) be a product intuitionistic fuzzy graph μ -density

$$D_{\mu}(G)=2\left(\frac{0.13+0.13+0.104+0.104+0.052}{0.2+0.2+0.16+0.16+0.08+0.08}\right.$$
$$\left.\frac{+0.052+0.039+0.0975+0.078}{+0.06+0.15+0.12}\right)=1.3$$

$$D_{\gamma}(G)=2\left(\frac{0.108+0.108+0.135+0.135+0.189}{0.24+0.24+0.3+0.3+0.42+0.35}\right.$$
$$\left.\frac{+0.1575+0.189+0.108+0.162}{+0.42+0.24+0.36}\right)=0.9$$

In Figure, For all subgraphs H of a PIFG G, D(H) < D(G). So G is balanced product IFG.

Example 3.2. Consider an IFG G = (V, E) such that V = {v_1, v_2, v_3, v_4}, E = {(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1), (v_1, v_3), (v_2, v_4)}.

Hence D(H) = D(G) for all non-empty subgraphs H of G. Hence G is strictly balanced IFG.

Example 3.3. Consider an IFG G = (V,E), such that V = {v1, v2, v3, v4}, E = {(v1, v2),(v2, v3),(v3, v4), (v4, v1)}.

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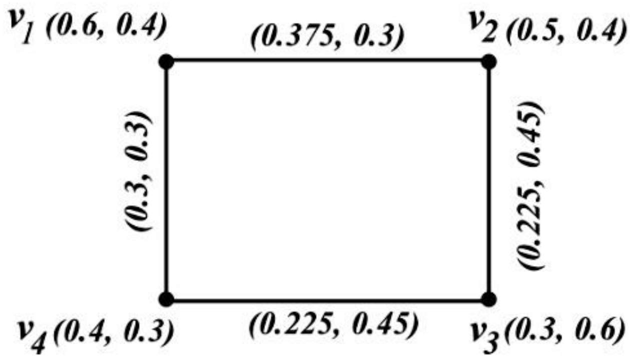


G

Hence D(H) ≤ D(G) for all subgraphs H of G. So G is balanced IFG. From the above graph easy to see that: μ2(u, v) = μ1(u)∧μ1(v) and ν2(u, v) = ν1(u)∨ν1(v)). Hence G is balanced but not complete.

PROPERTIES OF BALANCED IFG

Definition 4.1 The density of a product intuitionistic fuzzy graph G = (V, E) is D (G) = Dμ (G), Dγ (G), where Dμ (G) m is defined by

$$D_\mu(G) = \frac{2 \sum_{u,v \in V} (\mu_2(u,v))}{\sum_{(u,v)\in E} (\mu_1(u)\cdot \mu_1(v))}$$

Dγ (G) g is defined by

$$D_\gamma(G) = \frac{2 \sum_{u,v \in V} (\gamma_2(u,v))}{\sum_{(u,v)\in E} (\gamma_1(u)\cdot \gamma_1(v))}$$

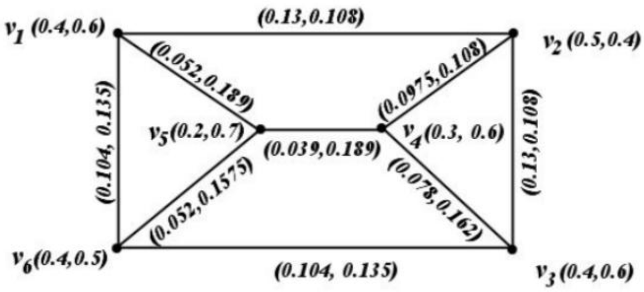
Definition 4.2 A product intuitionistic fuzzy graph G = (V, E) is balanced if D (H) < D (G) that is $D_\mu(H) \le D_\mu(G), D_\gamma(H) \le D_\gamma(G)$ for all sub graphs H of G.

Example 4.1 Consider G = (V, E) be an intuitionistic fuzzy graph as in Figure 3.

$$\mu\text{-density } D_{\mu}(G)=2\left(\frac{0.24+0.16+0.1+0.15+0.4}{0.24+0.16+0.1+0.15+0.4}\right)=2$$

$$\gamma\text{-density } D_{\gamma}(G)=2\left(\frac{0.12+0.16+0.32+0.24+0.08}{0.12+0.16+0.32+0.24+0.08}\right)=2$$

Theorem 4.1 A complement of a strong product intuitionistic fuzzy graph is balanced.



Proof: Let G = (V, E) be a strong product IFG and G = (V, E) be its complement.

Let G = (V, E) represent a finished good IFG As a result, the degree of a vertex's closed neighbors is defined as the sum of the vertices and its own membership values, while the degree of a vertex's closed g neighbors is defined as the sum of the vertices and its own non-membership values. As a result, the degree of closedness between any two vertices will be one. G follows from the fact that IFG is a regular product.

CONCLUSION

Since we have studied mild balanced IFG on the union and sum of two IFG sharing one or more vertices, we can easily expand this research to other operations, such as product and composition graphs, etc. In this article, we present the concept of the balanced double-layered graph and specify the circumstances under which a fuzzy double-layered graph may be considered "balanced." Provided some concrete examples to back up our findings and definitions.

REFERENCES

1. Selvanayaki. S, Strong and balanced irregular interval valued fuzzy graphs, International Journal of Engineering, Science and Mathematics Vol. 6 Issue 2, June 2017

2. V. Nivethana, Mild balanced Intuitionistic Fuzzy Graphs, Int. Journal of Engineering Research and Application, Vol. 7, Issue 3, (Part -3) March 2017, pp.13-20

3. Kishore Kumar. P.K, New Concepts on Mild Balanced Vague Graphs with Application, Intern. J. Fuzzy Mathematical Archive Vol. 15, No. 1, 2018, 37-53

4. T. Pathinathan & M. Peter, “Balanced Double Layered Fuzzy Graph”, International Journal of Multidisciplinary Research and Modern Education, Volume 3, Issue 1, Page Number 208-217, 2017.

5. M. G. Karunambigai, Properties of Balanced Intuitionistic Fuzzy Graphs, ScieXplore: International Journal of Research in Science, Vol 1(1), 01–05, January–June 2014

6. C.-H. Wang. "An intuitionistic fuzzy set-based hybrid approach to the innovative design evaluation mode for Wang, C. (2016). An intuitionistic fuzzy set-based hybrid approach to the innovative design evaluation mode for green products. Advances in Mechanical Engineering, 8 (4). doi: 10.1177/1687814016642715

7. Sarfraz, Mehwish, Kifayat Ullah, Maria Akram, Dragan Pamucar, and Darko Božanić. 2022. "Prioritized Aggregation Operators for Intuitionistic Fuzzy Information Based on Aczel–Alsina T-Norm and T-Conorm and Their Applications in Group Decision-Making" *Symmetry* 14, no. 12: 2655. https://doi.org/10.3390/sym14122655

8. Dan S, Kar MB, Majumder S, Roy B, Kar S, Pamucar D. Intuitionistic Type-2 Fuzzy Set and Its Properties. *Symmetry*. 2019; 11(6):808. https://doi.org/10.3390/sym11060808

9. Karunambigai, M. & Shanmugam, Sivasankar & Kasilingam, Palanivel. (2014). Properties of Balanced Intuitionistic Fuzzy Graph. ScieXplore: International Journal of Research in Science. 1. 10.15613/sijrs/2014/v1i1/53859.

10. Rashmanlou, Hossein & Pal, Madhumangal. (2013). Balanced Interval-Valued Fuzzy Graphs. Journal of Physical Sciences. 17. 43-57.