

Soft Demapping Techniques for Performance Enhancement in Coded MIMO-OFDM Systems

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Abstract-Multiple input multiple output-Orthogonal frequency division multiplexing (MIMO-OFDM) is a viable solution for providing high data rate services in harsh channel environments. The optimum receivers for them are those based on the maximum likelihood criterion. However, they have a prohibitive complexity especially when channel dimensions are high and coding is employed. Zero Forcing (ZF) and Linear Minimum Mean Square Error (MMSE) receivers on the other hand provide practicable and low complexity solutions for detection, but require soft demappers to deduce the soft bits information contained in each of the received symbols. In this work, we present the ZF and MMSE receiver analysis of a bit interleaved and coded MIMO-OFDM system and propose a soft output demapper based on MMSE equalizer output to demap the information needed for viterbi decoding. A comparison of the proposed soft demapper with conventional soft demappers in literature show a significant performance improvement. We also noticed that it is more advantageous to apply the proposed demapper on a MIMO-OFDM system employing higher modulation schemes.

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I. INTRODUCTION

Multiple Input Multiple Output (MIMO) combined multicarrier technology with modulation high data provides rate transmissions resulting from spatial multiplexing gain (between spatially seperated antennas at both the transmitter and receiver) and mitigation of the effect of frequency selective distortion in a richly scattered environment. The state-ofthe-art multicarrier modulation techniques include orthogonal

frequency division multiplexing (OFDM) [1], filtered OFDM [2], universal filtered multicarrier (UFMC) [3] [4], filter bank multi-carrier (FBMC) [5] [6] etc. In this paper, we study the synergistic combination of MIMO and OFDM techniques. Wireless LAN systems such as IEEE 802.11n [7], IEEE 802.11ac [8] and cellular systems such as 3GPP-LTE are exemplary cases to which MIMO-OFDM has been applied. To improve error performance due to fast fading, forward error correction (FEC) coding and bit interleaving are incorporated which is referred to as bit interleaved coded and modulated (BICM) MIMO-OFDM systems. The optimum receiver for this system is the one based on maximum likelihood detection but is very complex to implement in practice as its complexity increases exponentially with the number of transmitter antennas and the constellation size [9]. A more practical alternative is to employ linear equalizers to compensate channel fading and then pass the output to a soft demapper to extract the soft bits before de-interleaving and subsequent decoding.

Conventional soft demappers have been derived based on the assumption that equalizer outputs can be modelled as random variables with Gaussian distributions. [10] [11] derived the soft bits for MMSE-MIMO system as a function of signal to interference plus noise ratio (SINR) of the equalized symbols.

In this paper, we derived a new soft demapper that estimates the variance of coloured noise as a function of the conditional mean of the equalizer outputs. This method would add a little more complexity to the conventional soft demapper by computing the



mean and variance of the equalized symbols but achieves significant performance improvement.

The remainder of this paper is organised as follows: Sec. II presents the MIMO-OFDM system model. Sec. III analyses the ZF receiver and Sec. IV analyses MMSE receiver with the derivation of mean and variance of the equalizer output. In Sec. V, we derived the proposed MMSE soft demapper and also give the expression for conventional demapper. Sec. VI presents simulation results to compare the proposed soft demapper with the conventional method and we finally conclude the paper in Sec. VII.

II. SYSTEM MODEL

The system architecture for a BICM MIMO-OFDM transmitter is illustrated in Fig. 1. The input data bits are convolutionally encoded, interleaved and modulated using Gray-label mapping to produce complex 2 m - M ary QAM modulation symbols, each corresponding to b1, b2,bm bits sequence. A demultiplexer is then used to create Nt spatial streams of the modulated symbols. Each of the stream is passed through an IFFT block for mapping across OFDM subcarriers. To form an OFDM block, a cyclic prefix longer than the channel delay spread is inserted to mitigate the effect of both ISI and ICI. The complex OFDM symbols given by X(k) = Xr(k) + Xq(k) are assumed to be independent and identically distributed with zero mean and unit variance. The total transmit power is distributed equally over the transmit antennas. Xi(k) represents the k th subcarrier symbol to be transmitted on the i th transmit antenna of the Ntx1 antenna dimensions. In the receiver side shown in Fig.2, Nrx1 antennas are used to receive signals each comprising copies of symbols from the other transmit antennas. To recover the original signal transmitted from the transmitter after timing and frequency synchronisation, the cyclic prefix is removed and then FFT is performed to convert the time domain signals to frequency domain symbols. The output of the FFT-block at the k th subcarrier is given by

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where $y(k) = [y_1(k), y_2(k),] T \in C Nrx1$ is a column vector of received data symbols. x(k)= [x1(k), x2(k), ...,] T is a also a column vector of QAM symbols transmitted across Nt antennas at the k th subcarrier with E[|xi| 2] = $\sigma 2 x \cdot n(k) = [n1(k), n2(k), \dots] T \in C Nrx1$ is a complex additive white Gaussian noise vector which is due to thermal noise at the receiver, the element of which are independent and identically distributed with zero mean and covariance $E[|nn*|] = \sigma 2 n$. H(k) is the NrxNt MIMO channel frequency response matrix of the k th subcarrier. The elements of H(k) are also assumed to be independent and identically distributed zero mean circularly (i.i.d)symmetric complex Gaussian with Phji = 1 and given by

$$\mathbf{H}(k) = \begin{bmatrix} h_{j,1} & h_{j,2} & \dots & h_{j,N_t} \\ \vdots & \vdots & \vdots \vdots \\ h_{N_r,1} & h_{N_r,2} & \dots & h_{N_rN_t} \end{bmatrix}.$$
 (2)

Expanding Eq.(1) yields

$$y_j = \sum_{i=1}^{N_t} (h_{ji} x_i) + n_j.$$
(3)

The above expression shows that each receiver antenna gets a copy of symbols transmitted from all transmit antennas. Consequently, the system suffers from multi-antenna interference in addition to channel fading, necessitating equalization to remove those effects.

III. ZERO FORCING RECEIVER

The zero forcing equalizer aims to cancel the channel frequency response on the symbol of interest and forces all interferences of other symbols from neighbouring antennas to zero. ZF weight matrix is given as:

$$\mathbf{W} = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \tag{4}$$

The output of the equalizer is then given by

$$\mathbf{w}^H(\mathbf{h}_i x_i + \mathbf{n}) \tag{5}$$

where z is the equalized vector. The equalizer output for the i^{th} stream (i th transmitter) in the k^{th} subcarrier can be expressed as:

$$\mathbf{y}(k) = \mathbf{H}(k)\mathbf{x}(k) + \mathbf{n}(k), \tag{1}$$



$$z_i(K) = \mathbf{w}_i(K)\mathbf{h}_i(K)x_i(K) + \mathbf{w}_i(K)n(K)$$
(6)

where wi(k) and hi(k) are the i th row vector of W and i th column vector of H matrices in the K-subcarrier respectively. Eqn.6 can further be written as

$$z_i(K) = x_i(K) + v(K), \tag{7}$$

which is approximated by a complex Gaussian distributed random variable. The mean μzi can be estimated as E[zi] = xi and the variance of zi, can be computed as

$$\nu_{zi}^{2} = E[\nu\nu^{*}] = ||\mathbf{w}||^{2}\sigma_{n}^{2}$$
(8)

IV. MINIMUM MEAN SQUARE ERROR (MMSE) RECEIVER

The MIMO receiver separates the received vector into Nr parallel streams which can be

$$y = h_i x_i + \sum_{k \neq i} h_k x_k + n, \tag{9}$$

modelled as [12]

where y is a combination of symbol of interest xi which has gone through channel fading, ISI component from other transmitting antennas and the additive white Gaussian noise. A linear MMSE receiver on the other hand decouples the MIMO channel into Nt parallel streams representing estimated symbols from the transmitting antennas. The MMSE weight

$$\mathbf{W} = (\mathbf{H}^H \mathbf{H} + N_0 \mathbf{I})^{-1} \mathbf{H}^H$$
(10)

matrix is given as

where I is an identity matrix of dimension Nr. The output of the equalizer is then given by

$$\mathbf{z} = \mathbf{w}^{H}(\mathbf{h}_{i}x_{i} + \sum_{k \neq n} \mathbf{h}_{k}x_{k} + \mathbf{n})$$
(11)

where z is the equalized vector. The equalizer

$$z_i(K) = \mathbf{w}_i(K) \left(\mathbf{h}_i(K) x_i(K) + \sum_{k \neq n} \mathbf{h}_k(K) x_k(K) + \mathbf{n}(K) \right)$$
(12)

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output for the i th stream (i th transmitter) in the k th subcarrier can be expressed as

where wi(k) and hi(k) are the i th row vector of W and i th column vector of H matrices in the K-subcarrier respectively. The output of the MMSE equalizer (zi) is also approximated by a complex Gaussian distributed random variable

$$p(z_i|x_i, h_i) \sim CN(\mu_{zi}, \nu_{zi}^2).$$
 (13)

To compute the mean μ_{zi} using Eqn.(12), we have:

$$\mu_{zi} = E[z_i | x_i, H] = w_i^H (E[y | x_i, H])$$

= $w_i (x_i h_i - E[\sum x_k h_k | x_i, H])$ (14)

where α is given by

$$\alpha = E[\sum_{k \neq i} x_k / x_i]. \tag{15}$$

By substituting α as shown in appendix,

$$\mu_{zi} = w_i (x_i h_i - \sqrt{(N_t - 1)} x_i h_k)$$
(16)

where N_t is the number of transmitting antennas. For the variance $\nu_{zi}^2 = Var[z_i|x_i, H]$,

$$\nu_{zi}^2 = E[(z_i - \mu_{zi})(z_i - \mu_{zi})^* | x_i, H]$$
(17)

where $q = w_i(h_i - \sqrt{(N_t - 1)}h_k)$ from Eqn.(16). we know from Eqn.(11) that $z = w^H.y$ and also $w^H = R_{yy}^{-1}P_{xy}$ in [13]. Since, $E[z_i|x_i, H] = qx_i$, then

$$E[z_i x_i | x_i, H] = q E[x_i x_i]$$
(18)

Therefore,

$$\nu_{zi}^{2} = E[wR_{yy}w^{H} - 2q^{2}x_{i}x_{i} + q^{2}x_{i}x_{i}]$$

= $qE_{x} - q^{2}E_{x}.$ (19)

with mean μzi and coloured noise variance v 2 zi.

V. SOFT OUTPUT DEMAPPING

Since we can estimate the mean and variance of the equalized symbols given in Eqns.(8)(16)&(19), the LLR of the n th bit of the equalized symbols can then be computed from the conditional pdf of zi which is given as

$$p(z|x=\beta) = \frac{1}{\sqrt{2\pi\sigma}} exp\left\{-\frac{1}{2}\frac{|z-\beta|^2}{\sigma^2}\right\},\qquad(20)$$

where β is the amplitude of the transmitted symbol on the QAM constellation. The LLR function is given by [14] as:

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$$\wedge(b_n) = \log\left[\frac{p(b_n = 1|z)}{(p(b_n = 0|z)}\right].$$
(21)

By applying bayes theorem and assuming bits are equiprobable, bn can be re-written as

$$\wedge (b_n) = \log \left[\frac{\sum p(z|b_n = \beta_1)}{\sum p(z|b_n = \beta_0)} \right]$$
(22)

The above equation can be approximated using the max-log-map function which can also be expressed as min-euclidean distance function as [14]

$$\wedge(b_n) = \frac{1}{\nu^2} \left\{ \min|z_i - \beta_1|^2 - \min|z_i - \beta_0|^2 \right\}$$
(23)

CASE I: Conventional Soft Demapper The conventional soft demapper is implemented with the assumption that noise variance is the same in all the equalized symbols and expressed by [15] as,

$$\wedge(b_n) = \frac{1}{\sigma_n^2} \bigg\{ min|z_i - \beta_1|^2 - min|z_i - \beta_0|^2 \bigg\}.$$
 (24)

CASE II: Scaled Soft Demapper

Because of noise colouration as a result of equalization, the performance of conventional demapper above can be enhanced in OFDM systems [16] by appropriately scaling the soft demapper using the variance of coloured noise. This was derived with the assumption that zero forcing equalization is optimum for OFDM systems as individaul subcarriers experience narrow-band fading.

$$\wedge(b_n) = ||\mathbf{w}||^2 \left\{ \min|z_i - \beta_1|^2 - \min|z_i - \beta_0|^2 \right\}$$
(25)

CASE III: Proposed Soft Demapper

This demapper is proposed for MMSE receivers to take into consideration the estimated mean and variance of the received symbols. According to the derivations in Eqns. (16)&(19), the proposed soft demapper can be expressed as:

$$\wedge (b_n) = \frac{q_z}{\nu_z^2} \left\{ \min |z_i - q\beta_1|^2 - \min |z_i - q\beta_0|^2 \right\}$$

= $\frac{1}{1 - w_i(h_i - \sqrt{(t-1)}h_k)} \left\{ \min |z_i - w_i(h_i - \sqrt{(t-1)}h_k)\beta_1|^2 - \min |z_i - w_i(h_i - \sqrt{(t-1)}h_k)\beta_0|^2 \right\}$ (26)

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Note that this soft demapping approach has an increasing computational complexity with constellation size and transmit antennas therefore we further simplify the algorithm to maintain approximately the same complexity according to [17]. Consider a OAM constellation with dn as half the distance between the partition boundaries relative to bn for n > 1 (see appendix). Then the general simplified LLR function can be approximated as:

$$D_n \simeq \begin{cases} z_i & n = 1 \\ -|D_{n-1}| + qd_n & n > 1. \end{cases}$$
(27)

$$\wedge(b_n) = \frac{q}{\nu^2} D_n \quad n \ge 1 \tag{28}$$

By substituting the q and v^2 into Eqn. (28), the simplified n th bit function can be expressed as:

$$\wedge(b_n) = \frac{1}{1 - w_i(h_i - \sqrt{(t-1)h_k}} D_n \quad n \ge 1.$$
 (29)

Equation (29) represents simplified LLR function that takes into consideration the effects of ISI and coloured noise due to filtering.

VI. SIMULATIONS AND RESULT

In order to evaluate the performance of the proposed MMSE-based soft demapper for BICM-MIMO-OFDM systems, we simulate the conventional demapper in CASE I using both ZF and MMSE estimates and compare the performance with CASE II and CASE III. In the simulations, we used a 1/2 rate convolutional encoder with a constraint length of 7,

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Fig. 2. BER performance of 2X2-256QAM-BICM-MIMO-OFDM system

generator polynomial of (131, 171)8, and a random interleaver. The FFT size that produced the OFDM symbols was choosen to be 256 and the length of CP is 16. We used the channel model adopted by IEEE802.11 working group in [18] to generate the channel fading between each j th receiver and i th transmitter.

The output of a linear MMSE equalizer is an estimate of the transmitted symbol which is affected by coloured Gaussian noise and needs to be demapped into the component bits that made up the transmitted symbol. We derive a soft bit demapper with the approriate scaling coefficients q and v 2 in Eqn.27. To reduce the

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complexity involved in computing minimum euclidean distance which grows exponentially with the modulation order, we simplify the algorithm to maintain its computational complexity even with higher modulation in Eqns. (27)&(29) and apply it to 16QAM and 256QAM modulation levels, and 2X2 MIMO dimension.



Fig. 3. Complexity Comparison

BER versus Eb/N0 plots in Fig. 3 and Fig. 4 both show superiority of Case II (ZF- Noise Scaled) and Case III (Proposed MMSE soft demapper) over Case I (Conventional soft demapper) in all the modulation levels used. We noticed a more distinct power reduction by our proposed scheme compared to conventional demapper in 256QAM MIMO-OFDM than in 16QAM MIMO-OFDM e.g., for 2X2 MIMO-OFDM system at BER of 10-4, our proposed scheme shows an improvement of 3.5dB over conventional method in 256QAM against 0.8dB improvemnet in 16QAM. Fig. 5 compares the computational time of all the algorithms which indicates the computational complexity to give a base line for trade-off analysis.

VII. CONCLUSION

We have analysed linear ZF and MMSE receivers for BICM-MIMO-OFDM systems and derived a soft-demapper based on the conditional mean and variance of the equalizer output. In order to reduce the increasing complexity associated with increasing order of modulation for minimum euclidean distance metric, we approximated the soft demapper with



an expression that fairly maintains the same computational complexity for all practicable OAM modulation. A BER performance comparison of the proposed soft demapper, a ZF scaled demapper and a conventional soft demapper from the literature confirms superiority of the proposed scheme especially at higher modulation order. This justifies the viability of the proposed soft demapper for practical low dimensioned MIMO systems with high order modulation schemes.

APPENDIX A

PROOF OF α IN Equation(16)

$$\alpha = E[\sum_{k \neq i} x_k / x_i]. \tag{30}$$

by taking the square of both sides,

$$\alpha.^{2} = E[\sum_{k \neq i} x_{k}/x_{i}].^{2}.$$

$$= (N_{t} - 1)E_{x}/E_{x}$$
(31)

Since $E_x = 1$ (symbol),

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$$\alpha = \sqrt{(N_t - 1)} \tag{32}$$

substituting back in Eqn.(17), the result follows.

APPENDIX B PROOF OF (29)

Deriving Eq.(28) from Eq.(22)

$$\wedge(b_n) = \log\left[\frac{\sum p(z|b_n = \beta_1)}{\sum p(z|b_n = \beta_0)}\right] = \log\left[\frac{\sum \frac{1}{\sqrt{2\pi\sigma_{\nu}}} - \frac{(z - \alpha\beta_1)^2}{\nu^2}}{\sum \frac{1}{\sqrt{2\pi\sigma_{\nu}}} - \frac{(z - \alpha\beta_0)^2}{\nu^2}}\right]$$
(33)

For a 16QAM constellation, we have 4-amplituude (β) levels which are -3β , $-\beta$, β and 3β . By substituting the amplitude

$$\wedge (b_0) = \log \left[\frac{exp - \frac{(z - \alpha\beta)^2}{\nu^2}}{exp - \frac{(z + 3\alpha\beta)^2}{\nu^2}} \right] \quad z < -2\beta$$

$$= \log \left[\frac{exp - \frac{(z - \alpha\beta)^2}{\nu^2}}{exp - \frac{(z + \alpha\beta)^2}{\nu^2}} \right] \quad -2\beta \le z < 2\beta$$

$$= \log \left[\frac{exp - \frac{(z - 3\alpha\beta)^2}{\nu^2}}{exp - \frac{(z + \alpha\beta)^2}{\nu^2}} \right] \quad -2\alpha \le z > 2\beta$$

$$\wedge (b_0) = 4\beta \left\{ \frac{\alpha}{\nu^2} (2z + 2\alpha\beta) \quad z < -2\beta \\ \frac{\alpha}{\nu^2} (2z - 2\alpha\beta) \quad z > 2\beta \right\}$$

$$\wedge (b_1) = \log \left[\frac{exp - \frac{(z + \alpha\beta)^2}{\nu^2}}{exp - \frac{(z + \alpha\beta)^2}{\nu^2}} \right] \quad z < 0$$

$$\wedge (b_1) = \log \left[\frac{exp - \frac{(z - \alpha\beta)^2}{\nu^2}}{exp - \frac{(z - \alpha\beta)^2}{\nu^2}} \right] \quad z > 0$$

$$\wedge (b_1) = 4\beta \left\{ \frac{\alpha}{\nu^2} (z + 2\alpha\beta) \quad z < 0 \\ \frac{\alpha}{\nu^2} (-z + 2\alpha\beta) \quad z > 0. \right\}$$

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By approximating the piecewise functions above, $\Lambda(b0)$ and $\Lambda(b1)$ can be expressed as

$$\wedge(b_0) = \frac{\alpha}{\nu^2}(z) \tag{37}$$

$$\wedge(b_1) = \frac{\alpha}{\nu^2}(-|z|+2).$$
(38)

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